The source of cosmological constant
—wave mechanics of particles in curved space-time(Part II)

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(Dated: March 11, 2023)

In this article, we will see that the existence of cosmological constant is a natural possible result of the new wave equation, which has been established in the first part of this series. After analyzing various possible sources of cosmological constant, the most likely conclusion is drawn that the cosmological constant is not from the vacuum energy of matter, but from space itself, it is an inherent essential feature of space itself.

chinaXiv:202303.00133v1

A series of observational facts in contemporary astronomy support the existence of a very small positive cosmological constant in the universe. It is generally believed that the cosmological constant comes from vacuum energy. However, the huge gap between the calculated value of vacuum energy in quantum field theory and the actual astronomical observation value has long puzzled theoretical physicists and cosmologists, which is called the cosmological constant problem [1]. To truly understand and solve the problem of cosmological constant is still based on the development of basic physics [2]. Recently, theoretical physics has made new progress, a universal wave equation of particle in curved space-time has been established [3]. Starting from the new wave equation, we will see that the existence of cosmological constant is a natural possible result of the new wave equation.

The wave function of microscopic particles in curved space-time satisfies the equation [3]

$$il_P\frac{\partial\psi}{\partial\tau} = -\frac{l_P^2}{2\sqrt{-g}}\Box^2(\sqrt{-g}\psi) - \frac{1}{2}\psi, \tag{1}$$

where l_P is Planck length, the four-dimensional operator $\Box = \hat{e_x}\frac{\partial}{\partial x} + \hat{e_y}\frac{\partial}{\partial y} + \hat{e_z}\frac{\partial}{\partial z} + \hat{e_t}\frac{\partial}{\partial t}$ and $\Box^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial t^2}$, $g = |g_{\mu\nu}|$ is the determinant of the metric tensor. τ is called proper time and there is a relationship $d\tau^2 = -ds^2 = -g_{\mu\nu}dx^\mu dx^\nu$.

By separating the amplitude and phase of the wave function, we let

$$\psi = Re^{\frac{iS}{l_P}}, \tag{2}$$

where R and S are real numbers. Substituting formula(2) into equation(1), after calculations, let the real part be equal to the real part and the imaginary part be equal to the imaginary part, we obtain [3]

$$\frac{\partial R}{\partial\tau} = -\frac{1}{2}\left(R\Box^2 S + 2\Box R\cdot\Box S + \frac{2R}{\sqrt{-g}}\Box\sqrt{-g}\cdot\Box S\right), \tag{3}$$

$$\frac{\partial S}{\partial\tau} = -\frac{1}{2}(\Box S)^2 + \frac{1}{2} + \frac{l_P^2}{2}\frac{\Box^2 R}{R} + \frac{l_P^2}{2\sqrt{-g}}\Box^2\sqrt{-g} + \frac{l_P^2}{\sqrt{-g}R}\Box\sqrt{-g}\cdot\Box R. \tag{4}$$

Equations(3) and (4) are completely equivalent to wave equation(1). Under the classical limit condition, $l_P\to 0$, equation(4) becomes

$$\frac{\partial S}{\partial\tau} + \frac{1}{2}(\Box S)^2 = \frac{1}{2}. \tag{5}$$

Because four-dimensional velocity$\mathbf{U} = \Box S$, equation(5) can be rewritten as [3]

$$\frac{\partial S}{\partial\tau} + \frac{1}{2}\mathbf{U}^2 = \frac{1}{2}. \tag{6}$$

Taking four-dimensional gradient on both sides of equation(6), we get

$$\frac{\partial\mathbf{U}}{\partial\tau} + (\mathbf{U}\cdot\Box)\,\mathbf{U} = \Box\frac{1}{2}. \tag{7}$$

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Because of the relationship $\frac{d\mathbf{U}}{d\tau} = \frac{\partial \mathbf{U}}{\partial \tau} + (\mathbf{U} \cdot \Box)\, \mathbf{U}$, at last equation(7) can be written as [3]

$$\frac{d\mathbf{U}}{d\tau} = \Box \frac{1}{2}. \tag{8}$$

Both sides of equation(8) are multiplied by the static mass of particle m, we get

$$m\frac{d\mathbf{U}}{d\tau} = m\Box\frac{1}{2} = -m\Box(-\frac{1}{2}), \tag{9}$$

or

$$\frac{dP}{d\tau} = -m\Box(-\frac{1}{2}), \tag{10}$$

where $P = m\mathbf{U}$ is four-dimensional momentum. Equation(10) shows that under the classical limit condition($l_P \to 0$), the wave equation(1) returns to the dynamic equation of special relativity, and at low speed, equation(10) will further return to the dynamic law of classical mechanics.

supposing the particle velocity v is far less than the speed of light c, we then have $dt = d\tau$, equation(10) further returns to

$$\begin{aligned} \frac{dp^i}{dt} &= -m\nabla(-\frac{1}{2}), \\ \frac{dE_k}{dt} &= -m\frac{\partial}{\partial t}(-\frac{1}{2}), \end{aligned} \tag{11}$$

where $p^i (i = 1, 2, 3)$ and E_k are momentum and kinetic energy in classical mechanics respectively.

The gravitational force acting on a particle with mass m in a gravitational potential φ is

$$F = -m\nabla\varphi \tag{12}$$

Comparing formula(11) with formula(12), we get

$$\varphi = -\frac{1}{2} + C, \tag{13}$$

where C is a constant. Therefore, formula(13) shows that a natural result of wave equation(1) is that there must be a constant gravitational potential in space. Now let's discuss the two possibilities of gravitational potential value in formula(13).

(1) The first possibility, $\varphi = -\frac{1}{2} + C = 0$. The gravitational potential in space is zero everywhere, which is a mediocre situation and will not produce any new results.

(2) The second possibility, $\varphi = -\frac{1}{2} + C \neq 0$. If there is a non-zero constant gravitational potential $\varphi = -\frac{1}{2} + C$ everywhere in space, the Schrödinger equation of free particles

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi \tag{14}$$

should be modified to

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V_0\psi \tag{15}$$

where $V_0 = m\varphi$. As long as the gravitational potential φ is small enough to make $m\varphi \ll \frac{p^2}{2m}$, there will be no obvious contradiction between equation(14) and equation(15), and the Schrödinger equation(14) of free particles is still valid. Therefore, the second possibility is that the gravitational potential φ in space is a non-zero, very small constant.

We discuss the physical meaning of the second possibility. There is a very small gravitational potential that is constant everywhere in space, the question is: what generates this gravitational potential?

(1) First of all, a particle moves in space and feels the gravitational potential that exists everywhere in space, which cannot be generated by the particle itself. In other words, a particle cannot feel the gravitational potential generated by itself.

(2) Vacuum energy of matter? Vacuum state is a special state of matter particles. In the vacuum state, all particles are in the state with the lowest energy $\frac{1}{2}\sqrt{k^2 + m^2}$, and their energy density is [1, 4]

$$\rho_{\text{vac}} = g\int_0^{k_{\max}} \frac{d^3k}{(2\pi)^3}\frac{1}{2}\sqrt{k^2 + m^2}, \tag{16}$$

chinaXiv:202303.00133v1

where the factor $g = 2s + 1$ accounts for the degree of freedom of the matter particle and $k_{\max}$ is the momentum cutoff. Consequently, vacuum energy is a part of the energy of matter, that is, the energy of matter includes two parts: one part is the energy of particles in the excited states ρ_m^{e}, and the other part is the energy of particles in the ground state (the lowest energy state $\frac{1}{2}\sqrt{k^2 + m^2}$) ρ_{vac}, which can be expressed as

$$\rho_m = \rho_m^{\mathrm{e}} + \rho_{\mathrm{vac}} \tag{17}$$

If there is no any matter particle in a certain space, thus there is no any matter particle in its lowest energy state in this space. This space is called pure space, that is, the space where the vacuum energy of matter is zero. For a single particle moving in pure space, according to wave equation(1), formula(13) is still valid, the particle still feels the existence of a non-zero, very small constant gravitational potential. Therefore, the constant gravitational potential cannot be generated by the vacuum energy of matter particles.

(3) Excluding the first and second possibilities, this non-zero and very small constant gravitational potential can only be generated by space itself. That is, space itself must have a non-zero very small energy density ρ_s to generate gravitational potential, and the energy density does not change with time and space, i.e. ρ_s=const., which is the essential attribute of space. In other words, space itself has mass, and the mass density is a constant, which is an inherent basic attribute of space itself. The inherent energy density or mass density of space is a basic constant of space, and also a basic constant in physics. Like other physical constants, its value can only be determined through experimental observations.

Space itself has mass, which can make us understand general relativity more deeply. Einstein's general theory of relativity tells us that matter with mass can bend space, and how to bend is determined by Einstein's field equation, but why can matter bend space? there is no answer in general relativity. Now we can answer this question: because space itself has a mass, it is bent by the attraction of a massive object. At the same time, we also understand that because there is no interaction between charge and mass, charge cannot bend space. Therefore, we can never geometrize electromagnetic interaction like gravity.

Because space itself has an energy density of ρ_s=const., Einstein's initial field equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \tag{18}$$

should be modified to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G(T_{\mu\nu} - \rho_s g_{\mu\nu}) \tag{19}$$

where $(T_{\mu\nu} - \rho_s g_{\mu\nu})$ represents the total energy momentum tensor generated by pure matter and pure space itself. Because vacuum energy(ρ_{vac} in formula(17)) is a part of matter energy, the contribution of vacuum energy of matter is also included in the energy momentum tensor $T_{\mu\nu}$ of matter contribution.

Equation(19) can be rewritten into a familiar form

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda_s g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{20}$$

where Λ_s is the cosmological constant and $\Lambda_s = 8\pi G\rho_s$, the subscript s indicates that it is not from vacuum energy, but from space itself.

If the source of cosmological constant is really space itself, it is the essential characteristic of space itself, and has nothing to do with the vacuum energy of matter field(ρ_{vac} in formula(17)), in order to make the whole theory self-consistent, we must answer these questions: why does the vacuum energy of the matter field make no contribution to the cosmological constant and why the current vacuum energy density is so small? I will answer these questions in the third part of this series.

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chinaXiv:202303.00133v1